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da Costa, José Rodrigues; Mata, Maria Eugénia; Justino, David

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# Estimating the Portuguese Average Cost of Capital

*José Rodrigues da Costa, Maria Eugénia Mata  
& David Justino\**

**Abstract:** »Schätzung der durchschnittlichen portugiesischen Kapitalkosten«. In spite of the importance of having a figure for the domestic average Cost of Capital to base the estimates of the discount rates used in a number of long-term investments, the fact is that Portugal does not yet know with confidence its own value. Part of the answer might be attached to the number of profound impacts that affected and disturbed its Capital Markets during the Twentieth Century, in particular the break introduced by the Carnation Revolution in 1974. This paper translates both a test to the methodology necessary to make such an estimate under the Portuguese constraints, and also a first estimate of such a figure. From the daily data available for the quotations of shares listed in the Lisbon Stock Exchange, a time series of a comprehensive index is constructed covering on a weekly basis (Wednesdays) a time sample of 31.5 years, from January 1978 to June 2009. It also constitutes the first part of a 3-year project intended to study the entire Twentieth Century and to produce an estimate for the Cost of Capital comparable to the values included in the 2002 book "The Triumph of the Optimists" authored by Dimson, Marsh and Staunton. Although the output parallels traditional UK and USA figures, the Portuguese estimate for the Equity Return Premium is around 8%.

**Keywords:** cost of capital, equity risk premium, share index, risk-free rate, general equilibrium.

## 1. Economic Relevance of a Good Estimate for the Cost of Capital

There are today several important areas in which a country needs to know its domestic average Cost of Capital, the search for which has triggered a number of other studies in many countries, especially in the UK and the USA, due to their cultural environment and their availability of recorded historical data. But the success of the American economy in the Twentieth Century may suggest higher than normal annual rates and that may be misleading when extrapolated to other national economies. Therefore, some years ago three professors of the

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\* Address all communications to: José Rodrigues da Costa, Universidade Nova de Lisboa, Faculdade de Economia, Campus de Campolide 1099-032, Lisbon, Portugal; e-mail: jcosta@fe.unl.pt.  
Maria Eugénia Mata, Universidade Nova de Lisboa, Faculdade de Economia, Campus de Campolide 1099-032, Lisbon, Portugal; e-mail: memata@fe.unl.pt.  
David Justino, Universidade Nova de Lisboa, Faculdade de Ciências Sociais e Humanas, Avenida de Berna, Lisbon, Portugal; e-mail: djustino@presidencia.pt.

London Business School – Dimson, Marsh and Staunton (DMS) – published a large study, involving 16 countries (including Spain), that covers the whole Twentieth Century<sup>1</sup>.

This study provided, for the first time, a long-run perspective for the cost of capital in the world, but it also showed the diverse financial behavior of the sampled countries, as some of them had significantly smaller capital costs in comparison with the historical values for the USA or for the UK. Unfortunately, Portugal was not included in that study, a limitation that might lead some of our long-term decisions to be taken based on the extrapolation of the cost of capital from countries considered to be similar.

The only study that includes Portugal was conducted by Jorion & Goetzmann<sup>2</sup> but only reports the period from 1931 to 1996 and it suffers from three limitations. The authors had to make use of indirect sources of information for Portugal – especially the “International Financial Statistics” published by the IMF; dividends are not included in the equity returns estimates, and this may undervalue the estimate; finally, the risk premium is measured as the capital returns in excess of inflation, not the risk-free rate.

The purpose of this paper is precisely to close that gap via an historical analysis identical to the one conducted by DMS. Therefore, this is an investigation into the past of the Portuguese Exchange share market to uncover any potential average return that, if stationary, may be extrapolated into the future. It is not a forecasting exercise as performed by a number of scholars including, for example, recent works from Goyal and Welch (2008) or Ferreira and Santa-Clara (2009), where some past economic/financial variables are used to estimate the near future value of a stock return.

Two reasons dictated our option. The first one is an intention to make simply an historical survey and the second one comes from the fact that we are convinced that one of the main characteristics of any efficient market is that any method accurate enough to deserve the attention of investors is immediately destroyed by the subsequent decisions of those very investors. As Ferreira and Santa-Clara wrote “...to the extent that what we are capturing is excessive predictability rather than risk premiums, the very success of our analysis will eventually destroy its usefulness.” That is, unless a new and accurate methodology of forecasting is used only by its author – and for a limited period of time – any volatile future is impossible to forecast with certainty. So, only stable values unveiled from the past can be taken into the future without much risk.

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<sup>1</sup> “The Triumph of the Optimists: 101 Years of Global Investment Returns”, 2002. This same study became such an important source of information, both for scholars and for practitioners, that it has been updated annually ever since – and expanded later to a 17th country – thanks to a co-operative agreement established between those three authors and the Dutch ABN-Amro Bank.

<sup>2</sup> “A Century of Global Stock Markets”, NBER, February 2000.

The fact that this discount rate is not known with certainty in any market does not mean that one cannot make an estimate by observing the history of his own capital markets. That is the purpose of this paper in relation to the particular case of the Portuguese domestic economy.

## 2. Literature Review

### 2.1. Why CAPM Model

There is now a consensus that variability of share returns requires them, on average, to pay more to an investor than debt instruments with similar maturity as a compensation for uncertainty. That positive spread depends on the particular issuer under consideration, but for the whole share market of a country there is an average difference that is called the domestic Equity Risk Premium (ERP).

There are different models that connect the individual spread of a particular share to a) the general mood of the whole market underlying that particular issuer and b) to other macroeconomic variables of the country, such as the annual GDP growth rate, some factors of the industrial sector involved, etc. However, any multivariable model requires the computation of a large number of parameters proportional to the number of explanatory variables used by the model. In this respect, the particular case of the Capital Asset Pricing Model (CAPM) is very attractive because it uses only one explanatory variable – the market average return – although this still requires the estimation of the three following parameters:

- i. the cost of risk-free debt ( $R_f$ );  
the particular level of risk of the issuer ( $\beta_i$ )  $R_i = R_{free} + \beta_i \cdot (\bar{R}_{market} - R_{free})$ ;
- ii. the average risk premium ( $\bar{R}_{market}$ ).

Therefore, the selection of the Capital Asset Pricing Model is a simplification measure justified on the grounds of the level of accuracy that it is still available, and also by the widespread adoption of similar approaches by other scholars.

### 2.2. Historical Versus Future Returns

All the long-term decisions are forward-looking computations and require the use of a discount rate that shall be valid for the future, not a Cost of Capital that was in place in the past. However, this single value is not observable in the market and there is as yet no known model that quantifies such a value for any

time-frame ahead. There are many studies that have developed methodologies for predicting the equity premium<sup>3</sup> but their results do not hold for the 1990s.<sup>4</sup>

Mehra and Prescott (1985) used an 1889-1978 data base for GDP and Consumption in the USA and concluded that Arrow-Debreu asset-pricing models could not explain the high (American) equity risk premium at the same time as the small average risk-free return that was historically observed. Rietz (1988) re-specified that model for a frictionless pure-exchange economy and solved the puzzle in capturing the effects of (possible) market crashes by abandoning the hypothesis that consumption growth rates are symmetric about their mean (and fall above their mean as often as they fall below). Reasonable degrees of time-preference and risk aversion were found, provided that plausible severe crashes are not too improbable in the long-term analysis.

Barro and Ursúa (2008) went into full annual data on Consumption for 22 countries (including Portugal) to detect crises, as this is the variable “that enters into usual asset-price equations”. To enlarge the sample they also used GDP for 35 countries (maintaining Portugal). For samples that start as early as 1870 (as is the case for Portuguese GDP estimations) a peak-to-trough method was used for each country to isolate economic crises (defined as cumulative declines in Consumption or GDP by at least 10%) and 87 crises for consumption and 148 for GDP were discovered. This led to the conclusion that 3.5 years was the average duration for disasters, having a mean of 21-22% declines, under a coincident timing both in Consumption and GDP. The conclusion is that their model accords with “the observed average equity premium of around 7% levered equity”, after assuming that 3.5 is the coefficient of relative risk aversion.

Therefore, it is little wonder that people began to look again into the past in order to estimate that historical cost, hoping that the future would not be much different from that observed past<sup>5</sup>. But this raises a number of problems, in particular, nothing guarantees a smooth replication of the past in the long-term future. And most countries did not accumulate enough information about their past – especially the distant past – to produce reliable estimates of such realized cost.

These limitations led the whole financial industry to base their estimates for the Cost of Capital on the single historical analysis that has been conducted by Ibbotson Associates for the US market, since this country recorded a time series that runs uninterrupted from the beginning of 1926. Alternatively, the industry turned to the UK market, where Barclays Capital and Credit Suisse First Boston both produced historical estimates for the British Cost of Capital from a series that starts in 1919. The similarity of all three final results led most

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<sup>3</sup> Fama and Schwert (1977; 1981); Rozeff (1984); Keim and Stambaugh (1986); Campbell and Schiller (1988 a, b); Fama and French (1988; 1989).

<sup>4</sup> Lettau and Ludvigson (2001) and Schwert (2002).

<sup>5</sup> Campbell and Thompson (2005), Hillebrand, Lee and Medeiros (2009).

capital budgeting, fund management practice, and regulators' decisions to be made traditionally from that much known American Equity Return Premium of (around) 8.5% p.a.<sup>6</sup>

Unfortunately, and more recently, this single "anglo-saxon" value became suspicious after the arguments coming from different grounds. On the one hand, both DMS (2002) and Goetzmann (1999) conducted historical estimates for some countries (including the US market) using data covering the entire Twentieth Century and obtained not a single common historical rate, but a wide range of different domestic average share returns, some of them significantly far from the US value, even after considering the impact of the various currencies involved. On top of that, Schwert (1998) noted that, extending the US data base backward to the beginning of the Nineteenth Century (a time series ca. 200-years long), the American average Cost of Capital becomes much less than the traditional Ibbotson result, a fact that may indicate that the risk premium may be non-stationary, and, if so, the future may be different from the observed past. Finally, an Equity Return Premium of 8.5% is too large to be compatible with both the rate of long-term economic annual growth of any economy (estimated to be around 2%) and the level of risk aversion normally accepted for an average investor (exponent  $\gamma$  between 1 and 2).

In relation to the first criticism, such a time variability of the Cost of Capital within one single country or among different countries makes sense, since markets inevitably comprise human beings and it is known that their mood does change in response to the economic and social conditions surrounding that market. That is, human reasons may determine, now and then, an adjustment of the equity spread demanded by investors to take the risk of price volatility in tandem with those same environment changes.

But why are the Ibbotson and DMS estimates for the US market (8.5%) so much larger than the Schwert estimate (4% p.a.) for the same country? The answer comes from what is now called<sup>7</sup> "survival bias": investors demand an Equity Return Premium not only to cope with the variability of stock returns, but also to compensate them for the potential total loss due to rare but catastrophic crises that are always possible, as recorded in the history of any country. That is, although people only require an extra payment to invest in volatile shares, because shares also suffer from a kind of "credit risk", the total premium must be large enough to pay for both sources of risk.<sup>8</sup>

Indeed, during the entire Twentieth Century the USA was lucky enough to avoid the great turmoil that affected, for example, Russia – two revolutions – or

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<sup>6</sup> This is the value indicated on the most recommended book, "Corporate Finance" by Richard Brealey and Stuart Myers in its successive editions.

<sup>7</sup> Brown, Goetzmann, and Ross (1995).

<sup>8</sup> As Jorion & Goetzmann (2000) stated: "To the extent that the event causing the break was anticipated, the market seems to have been able to gauge the gravity of the unfolding events. Price declines before breaks is consistent with increasing demand for risk compensation for a catastrophic event", page 14.

Germany and Japan – collapse of their economies after the loss of world wars and/or of invasion by foreign armies. On the contrary, the US economy developed throughout the same years in a rather smooth manner, in spite of some “minor” crises that were observed here and there, such as the Great Depression of 1929 and the two oil shocks of the 1970s and 1980s. In any case, the American financial market never closed for extended periods, there was no major nationalization affecting large sectors of that economy, and there were no significant social events affecting the nation. On the contrary, in the previous century there were the 1812-15 War with the British empire and the major Civil War (1861 to 1865), which brought vast devastation to some important regions of that country, and these may explain that, extending the time series to include these previous 100 years, the average equity return becomes closer to the observed values in those other more unstable countries.

Under this interpretation, the high Equity Return Premium observed during the Twentieth Century in the US market would be the result of a pessimistic view of the average American investor during those recent 100 years that required them to demand a compensation to cover the expected volatility of returns plus the risk of a potential total loss. But, in the Nineteenth Century, the realized Equity Return Premium already incorporates the actual losses of those catastrophic years, and this could explain the much lower premium found ex-post.

As to the second criticism, there are two economic models that suggest that the 8.5% Equity Return Premium estimated for the US market must be too large a figure:

- i. Gordon’s constant growth model<sup>9</sup> for corporations allows us to estimate the Equity Risk Premium from

$$ERP = g_{real} + \frac{D_1}{P_0} - r_{real}$$

where  $g$  is the constant growth rate of the company, and  $D/P$  is the percent return obtained from the amount  $D_1$  of (next year) dividends received from a share currently priced at  $P_0$ , and  $r$  is the cost of debt. For annual dividends on the order of 3% to 4% of  $P$  and a real return of long-term debt of around 2%, an annual Equity Risk Premium of 8.5% requires a growth rate larger than 6% p.a.,<sup>10</sup> which can only exist while a company is still in its infant stages of development<sup>11</sup>. But even that model is over-optimistic since it assumes a constant rate  $g$  forever. For a more realistic model with rates of growth

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<sup>9</sup> Gordon, Myron J. (1959). “Dividends, Earnings and Stock Prices”. *Review of Economics and Statistics* 41: 99-105.

<sup>10</sup> See Annex II.

<sup>11</sup> Otherwise – if unabated – that company would, sooner or later, become larger than its surrounding economy. Note that the consensus is for a long-term growth rate of any economy of about 2% p.a.

decreasing from an initial high value, as the company matures, the discount rates of future dividends must be less, meaning a smaller Equity Risk Premium than under the constant  $g$  model;

- ii. Consumption-based asset pricing seeks to estimate the Equity Risk Premium from economic theory, as that premium ought to be the expression of the risk aversion of investors, since the securities' extra returns are the price of deviating consumption from today into the future. The appropriate measure of the risk of investing in volatile assets is to assess the impact of that investment on the riskiness of future consumption. This leads us to the recognition that the key to investment risk is the correlation between asset returns and consumption variation: the higher that correlation the more risky are those assets because these investments pay off more to savers precisely when consumption is already high, and vice-versa:

$$\text{Risk Premium for an Asset} = \gamma \cdot \rho \cdot \sigma_{\Delta C} \cdot \sigma_R$$

where  $\gamma$  is the average investors' risk aversion and is the correlation between the percentage change in consumption  $\Delta C$  and the asset return  $R$ . Once again, for normal values of these four parameters<sup>12</sup>, the Equity Risk Premium obtained would be about 100 times lower than the empirical findings. Any accommodation based on accepting values of  $\gamma$  much larger than the classical range<sup>13</sup> is not possible because that would require extremely large real interest rates, which were never found in any market for a prolonged period of time.

It has been suggested that this consumption model might be either too conservative or too rational. As to the conservative side, Campbell and Cochrane (1997) proposed a change in the utility functions such that "as consumption drops toward the accustomed standard of living  $X$ , people become more risk averse because they are less willing to accept further declines in consumption". That is,  $\gamma$  is not constant and can become much larger than in classical models, especially when consumption falls and approaches that habit level.

On the psychological front, the development of Prospect Theory by Tversky and Kaheman (1992) allowed bringing some irrationality to the explanation of market behavior. This is what Benartzi and Thaler (1995) did: investors care more about the *returns* obtained than about the *value* of their portfolios. Since losses are particularly painful, some investors do not evaluate returns every day, but only at large intervals of time, avoiding the useless pains due to temporary losses. So, those investors evaluating their portfolios everyday require a large risk premium to hold shares instead of bonds, but those taking

<sup>12</sup> Normal orders of magnitude:  $\gamma=1$  or 2,  $\sigma_{\Delta C}$  is around 1% p.a.,  $\sigma_R$  around 20% p.a. and  $\rho \leq 20\%$ .

<sup>13</sup> Even if we are slightly more generous and accept  $\gamma$  around 10 and  $\rho$  around 40%, we find a risk premium of about 0.8% p.a.



large time intervals between evaluations have smaller probabilities of losses and require smaller risk premia. Therefore, there are many different risk premia in the market and such values may change over time according to the average time horizon of evaluation of investments.

In summary: since the known theoretical models are not yet able to supply reliable figures for future values of the ERP, we are left with the classic approach of estimating the future from the recorded past. However, the Ibbotson figure sounds exaggerated, not only for the US market, but also for other countries, suggesting that the best each country can do is to develop its own data base of stock returns and extend it backward as far as possible to improve the quality of the estimate.

However, this avenue raises some problems: Is the historical sample large enough to produce estimates falling within narrow confidence intervals? What if the Equity Risk Premium is itself variable?

This question of non-stationarity is crucial for historical estimates because the most simple analysis has implicit the assumption of stationarity. But there are reasons for suspecting that the true Equity Risk Premium may be changing over time, a fact that is in agreement with some tests that reveal statistically significant changes in market variability. So, unless we have a model of how the Equity Risk Premium varies over time, we may be misled by historical data and/or we obtain extremely large confidence intervals even from century-long time series.

Fortunately, there are also arguments in favor of giving some value to the results linearly extrapolated from those historical series:

- a. as mentioned above, one of the most comprehensive analyses was conducted by DMS (2002) for the entire Twentieth Century and covers 16 countries that represent more than 95% of the free-float market capitalization of all world equities at the start-2002; although the historical average of Risk Premium<sup>14</sup> varies among those countries, that premium is always positive and covers a range (arithmetic averages) between 3.2% and 10.6% p.a.;
- b. this large range of values is compatible with the different histories followed by the various countries in the sample, especially catastrophic events such as revolutions, nationalizations, etc. that plagued them;
- c. the average volatility of returns shown currently by most indices – around 15% p.a. – suggests that 100 years of history is not enough to reduce the uncertainty of the estimated average equity returns; that is, the confidence interval anticipated for those averages is still too large<sup>15</sup>; but is also compatible with the above range of values;

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<sup>14</sup> Relative to T-Bills, not to T-Bonds.

<sup>15</sup> Bradford Cornell: "...72 years' worth of data is not enough to measure the risk premium with sufficient precision to satisfy most investors ... even if it is assumed that the future is

- d. even if the return demanded from shares does vary over time, it is difficult to accept that it took some fixed value in the past but, from now on, has definitely changed; most likely, it has changed a number of times in the past – following the business cycle – without any up or down trend, and will do the same in the future; therefore, the average past of each country may not be very different from its future, and we can approach that historical average provided our data base covers a number of different business cycles;
- e. although there is a minimum rationality in price formation, recent studies have revealed the degree of irrationality present in this particular area of human behavior; so, it is possible that the high values of risk premium found in the past series are a simple reflection of some of those irrationalities and, unless we assume that mankind will change in the future and be fully rational from now on, we cannot reject those high ERPs.

All in all, it seems that our low level of knowledge of these matters still recommends the use of the past as the “least worst” predictor of the future and that justifies that, every year, the Ibbotson Group publishes a number of studies updating the statistical information from a number of countries: *Ibbotson Associates – Stocks, Bonds, Bills, and Inflation Yearbook*. Also, since the publication of *The Triumph of the Optimists* in 2002, which covered 101 years of 16 countries, the London Business School and the ABN-Amro Bank have partnered to update those historical results every year and to extend that type of analysis to a 17th new country. Finally, Prof. Damodaran also maintains a page on his website where some statistical data are also accessible, in particular the historical cost of capital.<sup>16</sup>

### 3. Constraints of the Portuguese Capital Market

#### 3.1. Liquidity Constraints in the Stock Exchange

Most share markets in small countries show low levels of liquidity, due mostly to their reduced economic dimension that translates into a small number of large domestic corporations that are listed, in parallel with insignificant foreign investors’ interest in all domestic companies. However, the situation in Portugal during the 30-plus years under analysis shows some additional constraints stemming from the following:

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like the past, the estimates are so imprecise that it is not clear what the risk premium has truly been in the past”, page 44.

<sup>16</sup> <<http://pages.stern.nyu.edu/~adamodar/>>.

- a. by tradition, the largest source of funding of our domestic businesses is bank credit, not shareholders' capital; the banking segment has always overshadowed the capital market in this country;
- b. during 1975 (following the Carnation Revolution in 1974), a large slice of the economy was nationalized – implying that the number of listed companies dropped significantly in one – and all overseas companies listed on the Lisbon Exchange ceased their operations following the political independencies (under leftist governments) granted to those overseas territories in that same year.

Adding to these limitations, the operations of our two domestic Exchanges were suspended on the 25 of April 1974, only reopening for share trading in March 1977<sup>17</sup>. All in all, these factors determined that, when trading in shares restarted, the number of listed companies was extremely reduced, none of them were large entities, and investors were very risk averse due to the traumas brought recently to them by the political and economic events following the Revolution.

Fortunately, in a few years it was possible to overcome most of these fears and to re-establish a “normal” capital market that was open to foreign investors and mature enough to accommodate the large privatization program the country executed during the 1990s. That “miracle” was even “bright” enough to call the attention of some other European Exchanges, a fact that led in February 2002 to the merger of our two domestic organized markets with the Euronext Group<sup>18</sup>.

### 3.2. Share Indices in Portugal

To the best of our knowledge, only very late in the Twentieth Century did Portugal start to compute share indices according to the standards of any other developed capital market. In fact, it was only in February 1991<sup>19</sup> that the Lisbon Stock Exchange launched a capitalization-weighted share index – then called the BVL-General Index<sup>20</sup> – with a time series dating back to the first trading day of 1988. This index is still being calculated once a day and released after the close of the trading session.

This initiative of the Lisbon Exchange was a response to the drawbacks arising from the two share indices that existed at the time (beginning of 1990s):

- a. the *Totta & Açores* Index was calculated and published every day by the local *Totta & Açores* Bank, but with a methodology for the selection of the

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<sup>17</sup> Although the order from the government was dated from the 4th of March, 1977, only on the 7th March was trading resumed. Meanwhile, the Porto Exchange remained closed until 1981.

<sup>18</sup> Later, in April 2007 this pure European Euronext Group merged with the NYSE Group to form the current transatlantic NYSE Euronext Group.

<sup>19</sup> Included in the Daily Bulletin of the Exchange, for the first time, on 25th February, 1991.

<sup>20</sup> This index was later renamed as PSI-General.

companies to be sampled and a set of rules for translating the corporate events in the daily index value – dividend payments, stock splits, etc – that were never made public, thereby casting a shadow of representativeness over the values of this index;

- b. the Bank of Portugal index (one single average value per month) was based on all the companies listed in the main market of the Lisbon Exchange, but it weighted each share price by the corresponding traded volume, a method that overvalued those securities having more trades, not those with more capital placed in the market.

The panorama before the Carnation Revolution of 1974 was similar to that just described, although both the Bank of Portugal and the Portuguese National Statistics Office were developing some indices to measure both the overall volume of trades and the average prices.

It is important to note that the reason for the Lisbon Stock Exchange to start that index time series only in 1988 is connected to the liquidity problems mentioned above: before 1988, a significant number of listed shares frequently showed zero transactions in a trading session, and when some trades took place, the number of securities transacted was very thin, raising questions about the economic representativeness of those agreed prices.

Currently the Portuguese Exchange computes some other share indices, in particular some sectorial ones alongside a short index comprising only 20 companies. The reasons that led us to emphasize the BVL-General index in our analysis were the following:

- a. it is the largest and most diversified sample, as it uses all companies with shares listed in the main market, not only the 20 most “representative” ones as the most spoken PSI-20 index;
- b. it includes large and small corporations, therefore any potential size effect is diluted in the sample; this criterion excluded all sectoral indices from our choice;
- c. it corrects for dividend payouts of the sampled shares, a feature that the PSI-20 index does not include;
- d. most important, it is the longest time series available: against a base date of 5/Jan/1988 for this index, all the sectoral indices start at the beginning of 1991, and the PSI-20 index starts on the very last day of 1992.

### 3.3. Risk-Free Interest Rates

Although it is understood that an Equity Risk Premium (ERP) measures the average excess return demanded by investors to take the uncertainty of the subsequent rewards obtained from those equities, there is no consensus about the maturity of the debt instruments that should be used as that reference basis: a short-term or a long-term rate?

At first sight, long-term rates would be preferable as they also incorporate a premium for the long maturity of the credit: it compares similar alternatives for funding new projects. However, long T-Bonds also suffer from high volatility of returns due to the variability of the interest rates in the market – which includes the effect of inflation variation – whereas short-lived T-Bills are much less vulnerable to the current price of money, and inflation is always factored in when each new issue is placed in the market.

This justifies the frequent double disclosure of Equity Return Premium – excess above T-Bills and above T-Bonds – adopted in a number of countries. This is important because, in the history of every country, there are periods of high unanticipated inflation rates that justify long periods with negative returns from long-term bonds if previously issued with low coupon rates. Those negative returns may mislead us when subtracting the inflated average equity return from such negative debt returns, leading to an excessively large ERP.

It is also not clear which level of credit risk imbedded in that interest rate can be accepted in a real case: the cost of money for operations with a Central Bank (or a National Treasury) or the rates of the Interbank Money Market, which still have some residual credit risk included.

Fortunately, the limitations of the Portuguese money market simplified our decisions in this regard:

- a. because the Portuguese T-Bills were created only in 1985 and that market was closed temporarily from 1998 to 2003, we were forced to exclude this source of information;
- b. during the first years of the period under analysis, the interest rates defined by our Central Bank resulted from the economic policies of the government of the day, not from any considerations of monetary policy; obviously they were also excluded;
- c. although in the time window analyzed our Interbank Money Market offered rates for a wide range of maturities – from overnight credits to one-year operations – the clear majority of the liquidity in that market was always concentrated in the short-term end of that spectrum.

Therefore, we excluded the T-Bills source, opted instead for the Interbank Money Market<sup>21</sup> (IMM) rather than Central Bank rates, and used only Overnight rates<sup>22</sup> (O/N). Note that, in a number of countries, the risk-free rate is “borrowed” from the issuing rate of one-month T-Bills, but we assumed that the difference between the overnight and those one-month rates were always

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<sup>21</sup> So our “risk-free rates”, although very short-term, involve some residual credit risk.

<sup>22</sup> But some details are in order at this moment:

- i) when, after July 1993, the domestic market began to offer three Overnight rates – for the same day, for the next day and for two days ahead – we selected the same day O/N case;
- ii) but during some years that shortest term segment included 24-hour to 72-hour maturities, without any distinction between the three cases.

significantly less than the estimation error of the average rate of return of shares.

Finally, it must be said that, due to the closing of the Portuguese IMM market at the end of 2008, we continued our time series of risk-free rate toward 2009 using the EONIA daily rate as the representative Portuguese short-term interest rate.

### 3.4. Long-Term Memory of Defaults

The confidence in Portuguese treasury securities as risk-free assets deserves some additional comments. Although it might be thought that treasuries are risk-free assets because they represent a governmental commitment for the near future, historical events in Portugal during the Nineteenth Century explain why our treasuries cannot be used as such, at least for a significant part of the 1800s and the initial part of the Twentieth Century.

In fact, the confidence in our treasury debt instruments was very low during the second half of the Nineteenth Century, when the total amount of public debt increased dramatically every year, thanks to a large surplus of public expenditure over the simultaneous tax collection. The result was a low market price for their placement in all the European capital markets. That is, the Portuguese government of that time was forced to issue high nominal amounts of public debt but could only receive much lower cash amounts from the few investors available.

Such negative historical experience ended with the declaration of a bankruptcy in 1892 (decree of 13 June), when our government declared that it could not fulfil all the debt contracts signed with its foreign lenders, following the abandonment of the gold-standard just the year before (July 1891). That default meant that all amortizations of treasuries were suspended, and the country would pay only 1/3 of all interests due under those contracts.

Of course, even before this partial default, a number of public sources already raised the fear about the Portuguese debt, as our credit rating was decreasing. Specialized newspapers disclosed information on the prices and rewards of our bonds and bills during that pre-bankruptcy period, and the returns demanded from those instruments increased in tandem with the mounting fears on the risk level of Portugal. The negotiations with the creditors after 1892 lasted for ten years, only leading to an agreement when a conversion of the loans was achieved in 1902 (law of 14 May).

The fact that capital markets do have memory means that investors always make their decisions based on a stock of accumulated knowledge, and the reality is that, between 1870 and 1913, the coverage of all types of news about Portugal, in the Times of London, reached an annual average of 102 reports. The ratio of good to bad economic news that was reported was 1.12%, exceed only by Russia among a sample of 16 following countries: Argentina, Brazil,

Canada, Chile, China, Colombia, Costa Rica, Greece, Egypt, Hungary, Japan, Mexico, Queensland, Sweden, Turkey, and Uruguay.<sup>23</sup>

Such a poor performance closed the doors of all international credit markets to Portugal, and only during the First World War could a loan be obtained from the UK, in spite of the special relationship between the two countries.

Note that in the Twentieth Century, the events in the 1960s and 1970s were also detrimental for investors' confidence in our capital market. Colonial wars began in 1961, damaging colonial exports from the zones under military pressure in the colonies. The effort to fight the war led to a different allocation of resources and to uncertainty and even adverse perspectives on the future for the colonial firms. The Carnation Revolution in 1974 brought a new political model based on socialism that was consecrated in the 1976 constitutional text. Very soon dozens of political parties were organized, most of them leftwing-oriented. The Communist Party performed an important role, while radical policies were adopted. Nationalizations of banks and other firms in the main economic sectors (insurance, large industries, and road transports) were carried out in 1975, while land expropriation in the large-property districts of Alentejo and Ribatejo was also executed. Transactions in the domestic Stock Exchanges were suspended in 1974 and the decolonization of all overseas territories left Portugal confined to her European territory plus the tiny Atlantic archipelagos of the Azores and Madeira.

Half a million Portuguese, who were living in the colonies, left for Portugal in 1974-75, representing an influx of over 5% to the Portuguese resident population, and requiring the Government to support them in their beginning of new economic activities in the country. On top of that, a severe economic recession in the country brought problems to our balance of payments. Export difficulties led to the currency depreciation in 1977, and an IMF stand-by agreement was required in that year.

Confidence in the prosperity of Portugal under the new democratic regime was based on the project of joining Europe, a hope that was fulfilled only in 1986. In fact, the 1972 free-trade agreement with the then EEC was, in 1976, transformed into an association treaty to which a membership application followed in 1977. However, the negotiations dragged on for years, and the 1980-83 recession demanded a second intervention of technical and financial help from IMF for the period 1983 to 1985.

Portuguese membership of EEC was finally achieved in 1986, together with Spain, in a decade that prepared Europe for further consolidation after the communist political regimes collapsed in the 1990s in Eastern Europe and Russia. The integration of the Democratic German Republic and the Maastricht

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<sup>23</sup> Magee, Gary, La Trobe University, "Investors, information and the British world, 1860-1913", paper presented at the EBHA, Milan, 2009. <<http://www.hnet.org/~business/bhcweb/annmeet/abstracts09.html#magee>>.

Treaty led to the European Union, while in Portugal, a social-democrat government began a privatization program.

Only at the end of the 1990s, when it became clear that Portugal would become a member of the first wave to enter the single European currency<sup>24</sup>, did our level of risk start to decrease, reaching the lowest levels for more than two centuries.

## 4. The Econometric Model

### 4.1. Stochastic Behavior

Although it is known that the stochastic model adopted by Black and Scholes (B&S)

$$dS = \mu S dt + \sigma \sqrt{dt} \varepsilon_t \quad \text{where } \varepsilon_t \sim N(0, 1)$$

does not correctly describe the random behavior of the share prices, we have assumed it to be accurate and simple enough to deserve our attention. Therefore, we took that model to justify the regression that we used to estimate the historical average return provided by the Portuguese shares during the 31.5 years of our sample<sup>25</sup>. That differential equation can be transformed into

$$d[\ln(S)] = \left( \mu - \frac{\sigma^2}{2} \right) dt + \sigma \sqrt{dt} \varepsilon_t$$

or

$$\ln(S_T) = \ln(S_0) + \underbrace{\left( \mu - \frac{\sigma^2}{2} \right) (T - t_0)}_{\text{deterministic term}} + \underbrace{\sigma \sqrt{T - t_0} \varepsilon_t}_{\text{random "noise"}}$$

and that suggests that the log values of the share prices – or of the index – follow some straight line with a slope given by  $\left[ \mu - \frac{\sigma^2}{2} \right]$ . It was this suggestion that led us to fit a line to the historical periodic log values of the Portuguese share index recorded during the years January 1978 to June 2009.

<sup>24</sup> Mind that the Euro was first introduced in January 1999 simply as a virtual currency, while all legacy coin and notes were introduced only in 2002.

<sup>25</sup> Some scholars have studied other Stock Exchanges and found that, for some of them, a Mean Reverting model of periodic returns could well describe their stochastic behavior. In our case and for the period analyzed, an adjusted autoregressive model AR(1) produced an  $R^2$  of only 0.07. However, this is not a universal view and some countries have even shown a change from such a mean reverting model to a random walk behavior. Due to that low  $R^2$ , and because there is yet no consensus in academia about this question, and since mean reverting is against the logic of market efficiency, we adopted the B&S model.



## 4.2. Historically Realized Return or a Trend Curve

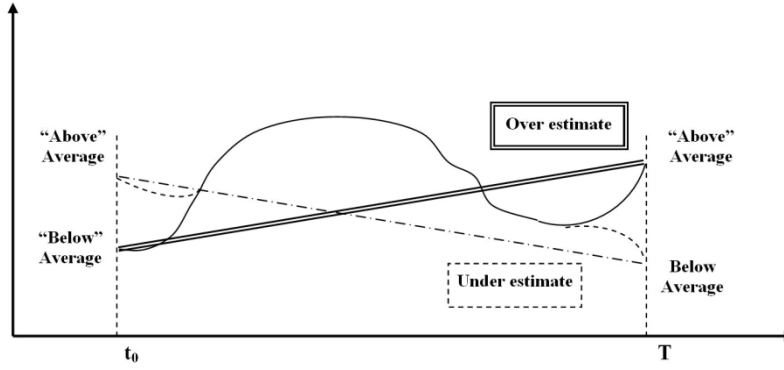
From a series of historical index values, the first impulse is to estimate the historical annual return from the arithmetic average of the  $n$  periodic returns observed during the entire sample<sup>26</sup>

$$\bar{R}_{historic} = \frac{\sum_{i=1}^n R_i}{n}$$

However, due to the use of logarithmic periodic returns, this average coincides with

$$R_{realized} = \frac{\ln(S_T) - \ln(S_0)}{T - t_0}$$

and that raises an important issue: this estimate does not take into account the particular evolution of the index between those two extreme dates. That is, it is irrelevant whether the initial  $S_0$  and/or the final value  $S_T$  corresponds to a peak (or to a trough) of a euphoric (or pessimistic) period, because the final result is always determined solely by those two extreme prices.



The consequence is that the estimate obtained from a short slice of history is crucially sensible to the starting and closing dates, particularly if either of those two prices is significantly deviated from the “average value” of the index at each time. This drawback can be minimized only for extended time series because:

- based on the B&S stochastic model, the deterministic term of the difference between the final and initial log prices grows linearly with the interval between those extreme dates, while the disturbing random term grows only with the square root of that same time span;

<sup>26</sup> We obtained an historical average of 15.9% p.a. (365 days), an annualised  $\sigma = 9.5\%$  and an error of 4.5%.

- b. therefore, even if the slope  $\left[\mu - \frac{\sigma^2}{2}\right]$  is small, this deterministic term will sooner or later dominate the random component, for intervals of time long enough to compensate for that reduced trend.

That is, according to the B&S stochastic model and for very long time series, most of the difference between  $\text{Ln}(S_T)$  and  $\text{Ln}(S_0)$  is due to the deterministic term – the slope of the line – because the random term affects that average only marginally. In our case, we have around 30 years of data and, for a common value  $\sigma = 15\%$  p.a. and for an average annual return of  $\left[\mu - \frac{\sigma^2}{2} = 10\%\right]$  p.a., we obtain:

- i. an accumulated return during the 30 years of  $10\% \times 30 = 300\%$ ;
- ii. and an accumulated volatility, in the same period, of  $15\% \times \sqrt{30} = 82\%$ .

These figures mean that we have only 5% of probability that the average annual return falls outside the range  $[300\% \pm 2 \times 82\%]/30$ , that is, it falls between 15.5% p.a. and 4.5% p.a.

Of course, the longer that time series, the smaller will be the impact of that noise term in the size of the confidence interval. It is this fact that only justifies the use of the realized return – the actual difference between the two extreme values – for estimates of the historical cost of capital when one has more than one century of continuous time series.

As a second best alternative we decided to estimate that average annual return adopting the logic of a curve fitting.

#### 4.3. Fitting an Exponential to the Historical Index Curve

Since there is a consensus that shares must provide investors with a certain expected periodic return – although disturbed by a permanent “noise” – one can anticipate that all share indices will tend to evolve through an “oscillating flight” around a “middle of the road” exponential line. The degree of deviations from that ideal trend curve will depend on the level of volatility  $\sigma$  around that average expected periodic return. Of course, things can be linearized by taking the logs of all index values.

That same purpose of fitting a straight line to the series of log values of the index comes also from the idea of estimating the parameters of the B&S model using the maximum likelihood method:

$$\text{Maximize } \prod_1^n \exp \left[ \frac{\left( (\text{Ln}(S_i) - \text{Ln}(S_0)) - \left( \mu - \frac{\sigma^2}{2} \right) \cdot t_i \right)^2}{\sigma_i^2} \right],$$

which leads, in essence, to the minimization of the square distances between the measured points of the log index and the estimated best-fitted line.

There are, however, some particularities that cannot be overlooked:

- a. the model assumes that the distribution of log prices around the straight line follows a Normal function with a standard deviation that grows over time, a fact that cannot be taken as granted because i) there are empirical

findings that suggest that  $\sigma$  is heteroscedastic, and ii) because there is frequently autocorrelation between successive returns (potentially, a residual characteristic of a certain mean reverting behavior<sup>27</sup>);

- b. there is a strong autocorrelation between log prices because any realized price for time  $t_i$  is almost entirely defined by the value of the same index for the previous moment  $t_{i-1}$ .

That is, the simplest form of the Ordinary Least Squares methodology for the line fitting has to be replaced by a more elaborated alternative that takes into account the variability of  $\sigma$  and the intense autocorrelation along the time series of log prices.

In summary, we estimated the average historical annual return using 31.5 years of the Lisbon Stock Exchange history and fitting a straight line to the log values of the share index covering that entire sample, not forgetting that this time series could show strong autocorrelation and some heteroscedasticity.

#### 4.4. Measurement Errors

As mentioned above, in a significant number of days during the 10 years from 1978 to 1987, many listed shares included in the index did not trade at all. In spite of that, when no agreed price existed, the daily index was computed using the average value between the Bid and the Ask prices as a second best alternative to such an equilibrium quotation. However, this decision introduced a source of error because we knew only the range – between the two values – where any equilibrium price could have been struck.

That is, extending the time series backward from the beginning of 1988 to January 1978 produces a set of index values with some measurement errors. One of them is due to this Bid/Ask spread and that can be easily estimated, but some others can only be mentioned:

- a. some companies only offered either a Bid or an Ask price, and that puts a limit to the range of potential equilibrium prices on one side only; additionally, it is not possible to make an estimate of an individual error for those days and for those shares where only one side is disclosed;
- b. even for those shares with an equilibrium price struck for the day, the very low volumes of shares actually traded raises the question of the representativeness of that quotation.

All of this suggests that the fitted straight line is estimated in the context of an input data (Y, X) where there are some measurement errors only in Y, not in X. But since those sampling errors do not affect the value for the estimated slope  $\beta$  but lead only to a confidence interval for that estimate a little larger than the

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<sup>27</sup> For our weekly sampled series, the autocorrelation of returns is 0.267 for one week lag, 0.260 for two weeks and 0.074 for three weeks.

real one, this specific subject of error does not affect our estimate and can be left for a future paper.

## 5. A First Sample of the Portuguese Market

### 5.1. Extending the Sample Backward

While still collecting data to construct a full series of index prices covering the entire Twentieth Century, we were tempted to take a first glimpse of the Portuguese market by analyzing the behavior of the BVL-General Index published since the beginning of 1988.

However, that short sample suffers from two important drawbacks: the base date – 5 Jan 1988 – happens to occur exactly in the aftermath of the October 1987 speculative crisis; additionally, the market is currently very depressed. Therefore:

- a. one may suspect that the initial value of the index might be above the long-term trend of the market due to the heavily “inflated” quotations observed during October 87; that is, the market was still returning to the “normal” levels during January 1988;
- b. the current index values might be too depressed because we are near the trough of the current financial crisis, which might be far below “average”;
- c. adding these two factors our estimate risks to be much below the average historical cost of capital trend.

We therefore felt the necessity to tap the domestic market before January 1988 in order to minimize the impact of that base date and also to enlarge the size of our time sample. In the end, we were able to add 10 more years to the existing 21.5-year series by computing a share index for the years 1978 to 1987, thus fabricating an uninterrupted 31.5-year long time series. In further studies, we plan to extend that index series toward the beginning of the Twentieth Century so that we can work with a sample of the same size as those used by other authors for other countries.

### 5.2. Sources of Information for the Period 1978-1987

Our main source of numerical data was the collection of Daily Bulletins published by the Lisbon Stock Exchange for the period 1978 to 1987, available from the Documentation Center of the Lisbon Exchange (now called Euronext Lisbon). Similar data from the Porto Stock Exchange were not used because this market remained closed until 1981 and also because, even after that date, it accounted only for a minor volume of the domestic secondary market.

Prices and quantities were collected only for Wednesdays in order to minimize the potential impact of the weekend effect, if any<sup>28</sup>. In the event a Wednesday happened to be a bank holiday, the following Thursday was taken, except if the market was also closed, in which case we took the best alternative available to represent that very same week.

For the purpose of constructing a share index, it was crucial to have information concerning all the corporate events that affected all the firms included in the index. Three main sources were used for this: the notices obligatorily published in the Lisbon Exchange's Daily Bulletin by all listed companies, the Annual Report of Activities for the relevant years of the Lisbon Stock Exchange, and some other statistical products published by the same Exchange.

Based on these sources, it was possible to adjust the weekly values of the index for all the corporate events that were detected for each of the firms included in the sample of the index. Note that this means in particular, that the reconstructed index measures the total return of the market – dividends plus capital gains/losses – not only price averages.

### 5.3. Tackling the Liquidity Constraints

#### *i) Agreed Quotations Versus Bid/Ask*

During the 10 years from 1978 to 1987, the trading mechanism in use at the Lisbon Exchange was one single auction per security per trading session. This Roll Call trading system called every security one at a time, and an equilibrium price was then searched by the brokers in order to maximize the number of shares that could be traded<sup>29</sup>.

As mentioned above, on a number of days, buy and sell orders for a number of issues did not clear at all, and no trades were possible on that particular day for that security. However, this did not preclude the market having information of the best buy and best sell offers entered into the auction crowd. Of course, these Bid and Ask prices did not represent an agreed quotation, but they were, nevertheless, an indication of the general level of prices that the market was attributing to those shares on those days.

Therefore, our option was to calculate the share index either with the single daily (agreed) quotation found in the auction – when that information was available – or alternatively with the simple average between the corresponding Bid and Ask prices. Also, in a few cases, we used only the Ask or the Bid if

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<sup>28</sup> Note also that, until May 3rd 1989, the Lisbon Exchange traded only four days a week. Reasons are mainly connected to the manual workload associated with Netting and Settlement of all past trades. Also, from the reopening in 1976 until September 20th 1978, there were only three sessions per week, on Mondays, Wednesdays, and Fridays.

<sup>29</sup> There were additional criteria for the case of more than one price of equilibrium maximizing the volume of trades.

that was the sole information disclosed in the Bulletin, and this in order to avoid computing the index with a very small pool of companies.

#### *ii) Weeks With no Price Information*

Unfortunately, for a few days, not even those isolated Bid and Ask prices were available, either because the security was suspended from trading (due to some corporate event), or because there was no interest in entering orders to the market. When such interruption extended only to one week or less, the index was calculated for that particular missing Wednesday assuming that the market would have priced that share exactly as in the previous week. But, for longer interruptions, that particular company was temporarily excluded from the index until its trades later resumed.

#### *iii) Market Without Quotations*

From February 1983 onward<sup>30</sup>, the Exchange market was divided into two segments:

- The so called “Official Market” – the main market – where the larger and “senior” companies could list their shares;
- and the “Market without Quotations” created to trade (but not list) some junior companies or the provisional certificates of shares (and bonds) issued by corporations already listed, while their final paper certificates were being printed and distributed among the investors.

These differences between the two segments suggested to us to use only the firms listed in the Official Market, but to take into account all the shares already issued by a listed corporation (if fully liberated) when computing the capitalization weight of that particular component of the index.

#### *iv) Selection of Companies to Compute the Index*

Due to the restricted number of corporations that had their shares listed in the main market of the Lisbon Exchange, we decided to use almost all of them, with the few exceptions of those so small or so infrequently traded that their contribution to the final value of the (capitalization weighted) index would be negligible.

In the later years of the 10-year period, a growing number of companies opened their capital to public investment and had their shares listed in our main market, most of them with so significant a size that they deserved their inclusion in the index from the very beginning of their listing. Table 1 indicates the number of companies in the index throughout the period.

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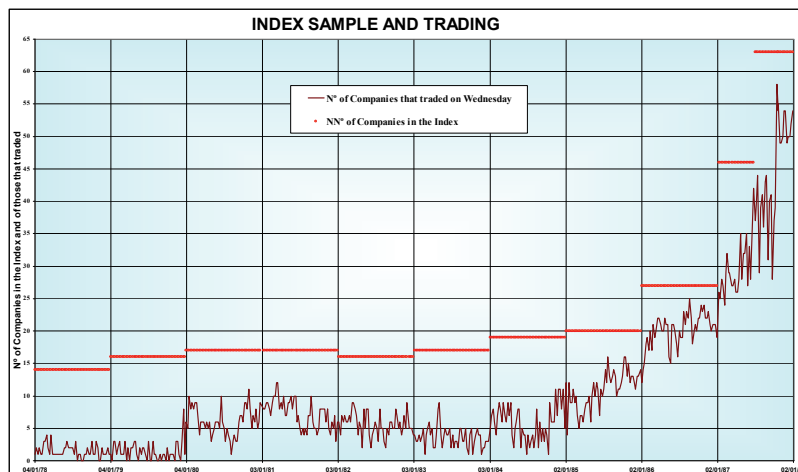
<sup>30</sup> See Daily Bulletin for 13 January 1983.

Table 1: The Number of Companies in the Index Throughout the Period

Year	1st Semester	2nd Semester
1978	14	14
1979	16	16
1980	17	17
1981	17	17
1982	16	16
1983	17	17
1984	19	19
1985	20	20
1986	27	27
1987	46	63
1988 <sup>31</sup>	Max = 128/Min = 80	Max = 154/Min = 123

Figure 1 compares, per week, the number of companies in the index sample with those that actually traded on the Wednesday representing that particular week. The panorama clearly improved from 1978 to 1987, which explains why we excluded the first year (1977) after the trade suspension imposed by the Carnation Revolution in April 1974.

Figure 1: Comparison, Per Week, of the Number of Companies in the Index Sample with Those that Actually Traded on the Wednesday Representing That Particular Week



<sup>31</sup> 1988 is the first year of the already existing BVL-General Index time series.

## 5.4. Heteroscedasticity and Autocorrelation

### *i) Autocorrelation*

It is frequently said that shares tend to exhibit autocorrelation in their time series, meaning that one day price *variation* is not fully independent of the previous day's performance and even from more distant dates. However, the autocorrelation that interests us is the interdependence of the *log prices* of the index over time, not the interdependence of the corresponding *log returns*.

Since our full time series of the index is made up of an extension of the BVL-General index series (computed daily from the beginning of 1988) with the new weekly sampled series for the 10 previous years (1978 to 1987), any autocorrelation estimate required the construction of a new time series with *only weekly* prices. This loss of information was compensated by the improved quality of the final estimate of the historical average annual return provided by the extended sample – about 50% more years.

In spite of the longer time interval between successive prices – one week rather than one day – one can forecast a strong autocorrelation from the very model adopted from the beginning:

- substituting, for simplicity sake,  $Y_t$  for  $\text{Ln}(S_t)$  in the B&S model

$$Y_{t+\Delta t} = Y_t + \left( \mu - \frac{\sigma^2}{2} \right) \Delta t + \sigma \sqrt{\Delta t} \varepsilon_t$$

this means that the next expected value  $Y_{t+\Delta t}$  tends to be close to the sum of today's value  $Y_t$  and a small deterministic term given by  $\left( \mu - \frac{\sigma^2}{2} \right) \Delta t$ ;

- therefore, the distance to the predicted value  $Y$  indicated by the linear regression line fitted to the historical evolution of the index –  $Y_{t+\Delta t} = a + b \cdot X_{t+\Delta t}$  – tends to be almost equal to the previous day case – distance to  $Y_t = a + b \cdot X_t$ ;
- this fact translates the idea that the (log) prices of the index do not distribute randomly around the fitted line, rather they tend to stick to the previous values, that is, they show a very strong autocorrelation.

In fact, the autocorrelogram of the 31.5-year weekly time series covering the interval January 1978 to June 2009 indicates the following values:

Time lag	$\rho$
1 week	0.998
2 weeks	0.995
3 weeks	0.992
4 weeks	0.989
5 weeks	0.985
6 weeks	0.982
7 weeks	0.978
8 weeks	0.974
9 weeks	0.971
10 weeks	0.967



## ii) Heteroscedasticity

Here we have two conflicting views. On the one hand, the B&S model suggests that uncertainty about the future grows over time as the variance of any estimate for a future price also grows with the time distance to that future date; and that may “facilitate” large future deviations around the exponential trend line. On the other hand, in the real world, prices never move far away from such a “middle-of-the-road” line, and empirical observations indicate that the volatility  $\sigma$  of the periodic returns is variable and there is some autocorrelation between successive period returns (against the random walk assumption of the B&S model).

Therefore, the fitted straight line was estimated via an OLS with potential heteroscedasticity and autocorrelation. We include a comparison with the results from the simplest OLS approach, as well.

## 6. Results

### 6.1. OLS Regression of Log Prices (Homoscedasticity and no Correlation)

The goal of this paper is to find the slope  $\beta$  of the fitted straight line, a coefficient that does not depend on any potential autocorrelation and heteroscedasticity that may exist in the time series:

Summary Output						
Homocedasticity & No Autocorrelation						
Regression Statistics						
Multiple R					0.902386694	
R Square					<b>0.814301745</b>	
Adjusted R Square					0.814188514	
Standard Error					76.03%	
Observations					1642	
ANOVA						
	df	SS	MS	F	Significance F	
Regression	1	4157.419	4157.419	7191.5316	8	
Residual	1640	948.0827	0.578099			
Total	<b>1641</b>	5105.501				
	Coefficients	Std Error	t Stat	P-value	Lower 95%	Upper 95%
Intercept ( $\alpha$ )	<b>-10.161919</b>	0.194501	-52.24605	0	-10.54341569	-9.7804219
X Variable ( $\beta$ )	<b>0.04795%</b>	5.65E-06	84.8029	0	0.04684%	0.04906%

From the two parameters ( $\alpha$ ,  $\beta$ ) of the line adjusted to the log values of the index, the corresponding parameters of the fitted exponential are:

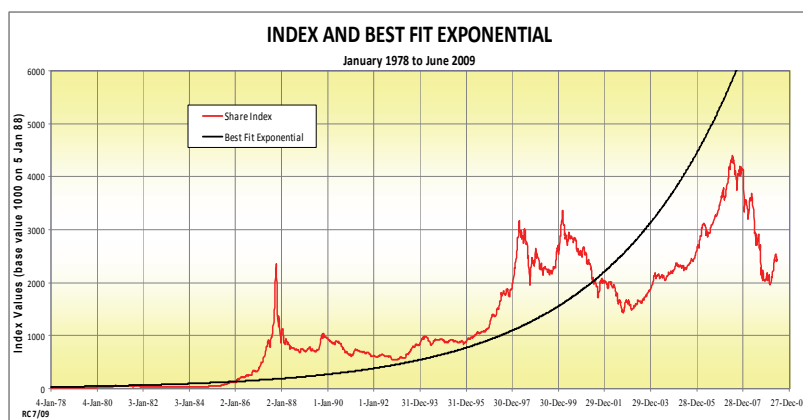
a. Average annual return<sup>32</sup>  $\beta = 365 \times 0.04795\% = 17.50\%$  p.a.  
Here, we took a 365-day year;

b. Initial value of the index (on 4 Jan 1978)  
 $\alpha = e^{-10.16192+0.0004795 \times 4 \text{ Jan } 78} = 33.12$  index points.

Here, the slope  $\beta$  is multiplied by the initial date of the new series.

Figure 2 superimposes the random evolution of the index to the best-fit exponential. It seems that, since the peak of quotations observed at the beginning of 2000 – the *dot.com* crash – our market could not recover enough to cross back over the exponential curve, and also the current crisis has even worsened the situation.

Figure 2: Superimposition of the Random Evolution of the Index to the Best-fit Exponential



This evolution of the last nine years may be a pure stochastic realization, but it may also be interpreted as:

- i. the Portuguese economy has been showing a difficulty in offering high rates of GDP growth in that same period, a fact that may already have been recognized by the investors as a structural change in our underlying economic model;
- ii. and/or, because the country joined the single currency area in the beginning of 1999, investors may now assume that, from that moment on, no major crisis is possible in our future that would impose a full loss

<sup>32</sup> Note that this  $\beta$  measures, in annual terms, the *continuous* rate of growth of the index (the trend), but the *relative* annual variation of that trend is given by  $\exp(\beta) - 1 = 19.13\%$  p.a. However, we are interested only in the first estimate, since it represents a geometric annual average due to the use of the log values of the index to be compared to the annualized geometric average risk-free rate.

- of their investments; if so, our Equity Risk Premium may now be in much lower levels than before;
- iii. note that, immediately after the reopening of the Stock Exchange in 1977 and also around the middle of the 1980s and 1990s, there was increasing confidence in the recovery of our domestic economy, a fact that was confirmed by the large number of companies that joined the stock market, and also by the tremendous success of the privatization program implemented from 1989 until 1999.

## 6.2. OLS Regression of Log Prices (Heteroscedasticity with Autocorrelation)

The existence of autocorrelation and/or heteroscedasticity in the regression does not change the estimate of the slope  $\beta$  of the fitted line, but it does change the confidence interval of that coefficient. However, one must consider it very likely that, during these 31.5 years, the market might have experienced different economic environments, that is, heteroscedasticity is most likely to exist. Also, the stochastic model adopted from the beginning suggests a strong autocorrelation in our time series of log values of the index. Therefore, an OLS estimate sensitive to these two characteristics was a must.

The 95% limits of confidence of the estimate are the following under the three different assumptions:

- i. homoscedasticity and no autocorrelation

95% Confidence Intervals (homoscedasticity)				
$\mu =$	17.50%		$S_{Jan78} =$	33.12
$\mu+ =$	17.91%		$(S_{Jan78})^+ =$	48.50
$\mu- =$	17.10%		$(S_{Jan78})^- =$	22.61

- ii. heteroscedasticity but no autocorrelation

There is not a great change in the accuracy of the  $\beta$  estimate, and therefore, of the confidence interval around the fitted curve.

95% Confidence Intervals				
$\mu =$	17.50%		$S_{Jan78} =$	33.12
$\mu+ =$	17.93%		$(S_{Jan78})^+ =$	50.24
$\mu- =$	17.07%		$(S_{Jan78})^- =$	21.83

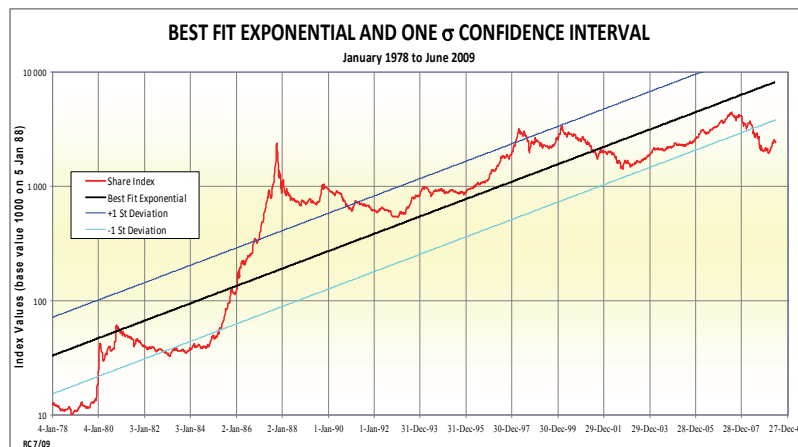
- iii. heteroscedasticity and autocorrelation

Autocorrelation worsens the quality of the  $\beta$  estimate:

95% Confidence Intervals			
$\mu =$	17.50%	$S_{Jan78} =$	33.12
$\mu + =$	18.70%	$(S_{Jan78})^+ =$	106.89
$\mu - =$	16.30%	$(S_{Jan78})^- =$	10.26

Under these more flexible estimates, the figure 3 compares the actual evolution of the index to the fitted straight line and places that evolution within the limiting borders of *one* standard deviation confidence interval.

Figure 3: Comparing the Actual Evolution of the Index to the Fitted Straight Line within the Limiting Borders of *one* Standard Deviation Confidence Interval



It is interesting to interpret the historical development of our equity market from this relative time evolution of our general share index within the estimated “confidence strip”:

- when the market reopened after the Carnation Revolution, the quotations were below the average, a fact that one would expect after the turmoil associated with that political, social, and economic revolution;
- the subsequent early recovery from the end of 1979 did not last long, due to our domestic economic crises, which could only be tackled after a second standby agreement was signed between Portugal and the IMF;
- the temporal coincidence of the termination of the 1983 crisis with the long-term recovery from the 1974-75 turmoil made possible the excessive quotations that ended with the October 1987 crash;
- the market then returned to more “normal” levels, but was again negatively influenced by the 1993 crises; but, in the beginning of 1988, the market was clearly above the average (more than one  $\sigma$  above), which

- explains the excessive base value of our first and longest daily share index – the BVL-General Index – observed on 5 Jan 1988;
- e. however, that world crises of 1993 was not enough to bring the index below average, probably due to the success of the ongoing privatization program that attracted many investors to our shares, especially the first large scale cash inflow from non-residents;
  - f. the *dot.com* peak at the beginning of 2000 may mark the end of this evolution around the trend line: after that peak, our market seems to be facing difficulties in crossing back over the “average” line, a fact that may have economic interpretations beyond simple stochastic explanations (as mentioned above);
  - g. in particular, the current trough of the index behavior places the market clearly away from “average” historical values; that is, although the graph may suggest that there must now exist a “strong force” dragging the index back toward the “middle-of-the-road” line, if our economy and our capital market did change in 1999/2000 and the expected return has dropped to lower values, the next world recovery may not bring our quotations back to levels close to that center line.

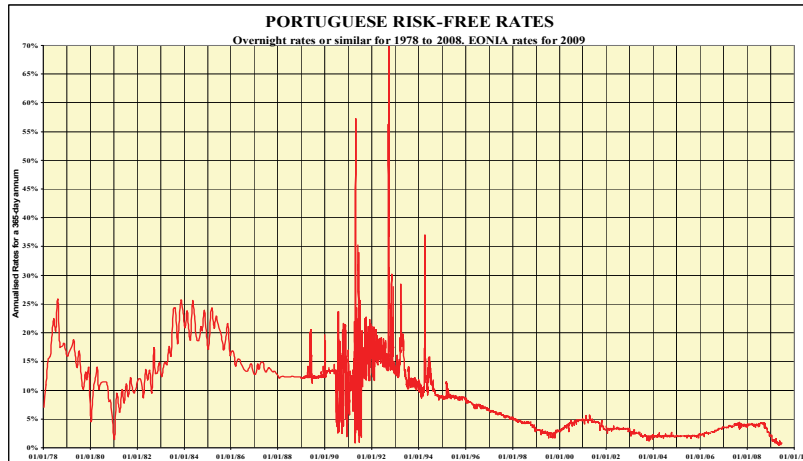
### 6.3. Risk-free Rate

As mentioned at the beginning, during the 31.5-year time window analyzed, the Portuguese money market suffered from severe problems of excess of interference from the domestic government and from significant lack of liquidity in all maturities segments other than overnight. This led us to select very short-term rates (instead of long-term ones) from the interbank market as the local risk-free cost of money. In detail:

- a. we did not consider any Central Bank rate, due to the excessive control by the government of the money market during a large number of the initial years of the sample used;
- b. we excluded any medium and long-term rates agreed in the IMM because most of the days recorded no operations at all in this market;
- c. in the earliest years, the very short-term segment included 24-hour, 48-hour and 72-hour loans, without distinguishing between those three different maturities;
- d. from the beginning of 2009 onwards, because the local IMM market was integrated into the much larger Euroland interbank market, we took EONIA as the substitute for the previous domestic overnight rate.

Figure 4 depicts the evolution of the daily (annualized) rates as published by the different sources of data.

Figure 4: The Evolution of the Daily (annualized)  
Rates as Published by the Different Sources of Data



The average rate along this 31.5-year series is a geometric average that is annualized assuming a 365-day year. But, since the input data were measured using different frequencies, it means that:

- when, during the years 1978 to 1988, we had only average monthly rates, we accumulated them using a year of 12 equal months and assuming that those published rates had been annualized through that same multiple;
- from the beginning of 1988 onwards, rates became available on a daily basis and the accumulation was computed assuming a 360-day year, as this was the tradition in our domestic money market;
- similarly for EONIA rates used for 2009, since these are reported on an Act/360 day count convention;
- however, the final geometric average for the whole 31.5-year window was annualized for a 365-day year because that risk-free rate is to be compared to an average annual equity return that also assumes a 365-day year.

The final average risk-free rate obtained is

$$R_f = 9.47\% \text{ p.a.}$$

#### 6.4. Equity Risk Premium Above Short-term Rates

From the geometric equity return and subtracting the geometric risk-free rate, the average Equity Return Premium relative to short-term rates is 8.03%.

Equity Return =	17.50%
Riskfree =	9.47%
ERP =	8.03%

## 7. Summary and Conclusions

Extending 10 years backward the pre-existing BVL-General share index time series that starts in the beginning of 1988, we adjusted a straight line to the log values of that index assuming both autocorrelation and heteroscedasticity for those prices. This is a rather detailed time series – 1652 points – since we sampled every week (as a rule, all Wednesdays) along the 31.5 years from January 1978 to June 2009.

The main result obtained from that curve fitting was the average annual (continuous) return provided by that market during that window of time: 17.5% p.a. (365-day year) with a 95% confidence interval of  $\pm 1.2\%$ . Note that this is a geometric average estimate, not an arithmetic one, as the underlying values are the log prices of the index.

Because our Treasuries did not exist during some of the sampled years – the case of T-Bills – or were very illiquid – the case of T-Bonds – the risk-free rate was estimated using the domestic Interbank Money Market (in 2009 we used the European reference EONIA): the geometric annualized average interest rate for overnight operations or similar for the same period of 31.5 years was 9.47% p.a. (again 365-day year).

These two figures led to an estimate of the historical Equity Risk Premium in Portugal above short-term risk-free rate of 8.03% p.a., a value of the same order of magnitude found in other countries, in particular, the traditional Ibbotson figure estimated from the US sample starting in 1926.

For the future, this historical average might be excessive because it might have resulted from an extra premium demanded by investors based on the negative experiences they had gone through recently and the lack of liquidity during the same first years after trading resumed in the Stock Exchange. And, in fact, the most recent 8 to 9 years of the index suggests a below average behavior.

The causes of this evolution may be a simple impact of the accession of Portugal to the European Union (in 1986), in particular the sharing of the European currency since its inception in 1999: investors, both domestic and from abroad, now demand a lower premium due to this European “seal of quality”. An alternative cause may be in the low pace of GDP growth of our economy during the same years, which is interpreted as a long-range effect upon the current value of our shares. Of course, simple stochastic behavior may also play a role.

This suggests additional studies in the double direction of extending even further the series toward the beginning of the Twentieth Century, and to include the measurement of the Equity Risk Premium in relation to long-term real rates – after discounting for inflation – in order to better gauge the real return obtained from long bonds.

An important dialogue may be established between macroeconomists and finance historians in looking for evidence from the large laboratory of experiences and facts that history makes available whenever long-term analyses are pursued. Stock Exchange variables are now a decisive topic for the global scholarly community and for theoretical paradigms of different schools of economic thought.

## Annex I

### Capitalization-Weighted Share Indices

#### *Rules of Computation*

- 1) The oldest Portuguese share index still being calculated is the BVL/PSI-General<sup>33</sup>, one which started the daily series on 5 Jan 1988 with a base value of 1000 points. This index includes all shares listed in the main market of the Lisbon Stock Exchange and weights each component according to the number of shares listed.

Everyday, a single value is computed based on the closing prices of all the shares included in the sample. Also, all corporate events affecting the price of any share beyond market sentiment are taken into account through proper adjustments either in the numerator or the denominator of the formula.

However, for dates before January 1988, there is nothing compatible with this index since the different series known, either never disclosed the methodology adopted to calculate the index or followed solutions not compatible to the above index.

Therefore, what the present paper does is to explain the solutions adopted to replicate as close as possible the methodology of the BVL-General index of the Lisbon Exchange for the period 1978-1987. This period is very relevant for future developments in this area of indices because:

- the base date adopted for the BVL-General index – beginning of 1988 – might still be somewhat influenced by the excessive speculation of the two previous years, which culminated in the spectacular crash of October 1987; that is, it might be “overvalued”;

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<sup>33</sup> It started as BVL-General in 1991, but was later re-baptised PSI-General, the name in use today.



- the Portuguese share market was in the 1980s still recovering from the “wounds” that followed the economic and social events of 1974, in particular the “suspension” of the Exchange operations for about two years<sup>34</sup>; if the methodology of the current index can be adapted to express the average market behavior during that 10-year time window, then it will be extended to the earlier periods in another study.
- 2) The source of information used to continue the index toward earlier years is the daily bulletins published in paper form by the Lisbon Stock Exchange and available at the Documentation Centre of (today called) Euronext Lisbon<sup>35</sup>. However, only one day per week was used, in order to reduce the computation burden. The selection led to the use of every Wednesday – except if it was a bank holiday – in order to minimize potential weekend effects upon the observed prices.
- 3) Since the index covers all shares listed in the main market of the Lisbon Stock Exchange, the sample had to be modified and the index adjusted whenever a company entered the market or left it:  
For the case of a new company entering the market, the index did not consider the first day of quotation, but added its capitalization only on the second day of the time series:

$$I_{t+1} = I_t \cdot \frac{\sum_1^n (Capitalization_i)_{t+1}}{\sum_1^n (Capitalization_i)_t} \quad \begin{cases} t = \text{first day of the new company} \\ t+1 = \text{second day of the new company} \end{cases}$$

where the denominator already includes the capitalization of the new company on the first day of trading. Of course, for the first day, the numerator excludes the capitalization of the new company in order to reduce the index variation to simple changes in the sentiment of the market.

$$I_t = I_{t-1} \cdot \frac{\sum_1^{n-1} (Capitalization_i)_t}{\sum_1^{n-1} (Capitalization_i)_{t-1}}$$

When a company left the market by any reason, the denominator excludes that company for the computation of the index on the first day already without the old company:

$$I_t = I_{t-1} \cdot \frac{\sum_1^{n-1} (Capitalization_i)_t}{\sum_1^{n-1} (Capitalization_i)_{t-1}} \quad \begin{cases} t = \text{first day of the new company} \\ t-1 = \text{second day of the new company} \end{cases}$$

- 4) This index measures the total return of the market including the yield due to cash dividends paid out by some of the companies in the sample. To include this impact of the dividend, the numerator of the formula was expanded to include the total amount of cash dividend outflow from that

<sup>34</sup> The Exchange closed for trading on 25/April/1974 and reopened for share trading only on 7/March/1977.

<sup>35</sup> After the Lisbon Stock Exchange joined the Euronext group of Exchanges, in February 2002, this is the legal name adopted by this Portuguese affiliate.

firm on the first Wednesday after the payment day. But, for all other days, only the sum of capitalizations without those paid dividends were used in the formula:

$$I_t = I_{t-1} \cdot \frac{\sum_1^n (Capitalization_i)_t + N_j \cdot D_j}{\sum_1^n (Capitalization_i)_{t-1}} \quad \begin{cases} t = \text{first Wednesday after divided pay - out} \\ t-1 = \text{the previous Wednesday} \end{cases}$$

- 5) Unfortunately, the market did not offer trades of shares for all listed companies on every day. That is, there was a frequent lack of quotations throughout the time window analyzed. However, the daily bulletin also discloses the best Bid and the best Ask prices after each particular auction<sup>36</sup>. Therefore, as a second-best solution, the following methodology was adopted:
- when no trades had occurred, the average between Bid and Ask was used as a proxy for the real traded price;
  - when only the Bid or the Ask was available, that single value was used as that proxy;
  - when none of the 3 values were present on a particular Wednesday, the value from the previous Wednesday was used;
  - however, if two or more Wednesdays lacking data occurred in a row, that company was temporarily excluded from the sample according to the rules mentioned above for new entrants and old companies leaving the market.

## Annex II

### Macro-Economic View of the Risk Premium

#### *Gordon's Model*

The DCF formula for a constant rate of dividends leads to an annual equity return of

$$R = g + \frac{D_1}{P_0}$$

So, for an annual cost of debt (Treasuries) of  $r$ , the equity risk premium (ERP) is

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<sup>36</sup> Note that the Stock Exchange used the Roll Call system of trading during this period, where each listed share was called once at a time and a single equilibrium price was discovered after confrontation of all the orders carried by the brokers for that particular asset. The prices of those orders closest to that equilibrium but already excluded from trading produced the daily Bid and Ask prices.

$$ERP = g + \frac{D_1}{P_0} - r$$

Note that this premium can be measured either in relation to T-Bills or T-Bonds, without a consensus yet established. For an inflation rate of around  $\pi$  and a real interest rate of debt of  $r_{\text{real}}$ , this implies a nominal cost of treasuries of about  $r_{\text{real}} + \pi$ , that is

$$ERP = g + \frac{D_1}{P_0} - (r_{\text{real}} + \pi)$$

But the difference ( $g - \pi$ ) measures the real growth rate of the company

$$ERP = g_{\text{real}} + \frac{D_1}{P_0} - r_{\text{real}}$$

Companies cannot grow forever more than the general economy surrounding it, for which a (real) long-run rate is estimated at a maximum of 2.5% p.a. Since the real return of long-term Treasuries is about that same size, it implies an Equity Return Premium relative to Bonds similar to the yield from Dividends. Therefore, a bond Equity Return Premium of 6% p.a. can only be compatible with a much larger  $g_{\text{real}}$  and that is only acceptable during limited time intervals. Of course, working with an Equity Return Premium relative to bills, that figure jumps to 8.5%, but the conclusion is the same.

The reality is even slightly worse because companies may begin with large rates of growth  $g$  but, that pace always tends to fall as the firm matures. So for the same  $D_1$  and  $P_0$ , the discount rate and the Equity Return Premium must be smaller than forecast from the standard Gordon formula. This means that the excess of Equity Return Premium suggested by the standard Gordon's formula is even more excessive.

That is, 3% to 4% Equity Return Premium over short-term Treasuries sounds excessive, much worse for an Equity Return Premium of 8.5% p.a. as advanced by Ibbotson.

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